

TWO-FLUID VISCOUS MODIFIED GRAVITY ON A RS BRANE

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Abstract

Singularities in the dark energy late universe are discussed, under the assumption that the Lagrangian contains the Einstein term R plus a modified gravity term R^α , where α is a constant. The 4D fluid is taken to be viscous and composed of two components, one Einstein component where the bulk viscosity is proportional to the scalar expansion θ , and another modified component where the bulk viscosity is proportional to the power $\theta^{2\alpha-1}$. Under these conditions it is known from earlier that the bulk viscosity can drive the fluid from the quintessence region ($w > -1$) into the phantom region ($w < -1$), where w is the thermodynamical parameter [I. Brevik, Gen. Rel. Grav. **38**, 1317 (2006)]. We combine this 4D theory with the 5D Randall-Sundrum II theory in which there is a single spatially flat brane situated at $y = 0$. We find that the Big Rip singularity, which occurs in 4D theory if $\alpha > 1/2$, carries over to the 5D metric in the bulk, $|y| > 0$. The present investigation generalizes that of an earlier paper [I. Brevik, arXiv:0807.1797; to appear in Eur. Phys. J. C] in which only a one-component modified fluid was present.

Keywords: modified gravity; viscous cosmology; Randall-Sundrum model

1 Introduction

There has recently been a great deal of interest on modified gravity theories in four dimensions, and what consequences they may have for the Big Rip singularity in the late universe (cf., for instance, the review of Copeland

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et al. [1]). The equation of state for the cosmic fluid is conventionally written as $p = w\rho \equiv (\gamma - 1)\rho$, where $w = -1$ corresponds to a vacuum fluid (or cosmological constant), $-1 < w < -1/3$ to a quintessence fluid, and $w < -1$ to a phantom fluid. The last-mentioned case predicts a Big Rip singularity in the late universe. Both quintessence and phantom fluids lead to the thermodynamical inequality $\rho + 3p \leq 0$, thus breaking the strong energy condition.

It is of interest to combine the modified gravity in 4D with the Randall-Sundrum model in 5D. We shall consider the RS II model, in which there is a single spatially flat brane situated at $y = 0$, surrounded by an AdS space [2]. Therewith one can analyze the relationship between the 4D Big Rip singularity and the corresponding behavior of the metric in the 5D bulk, $|y| > 0$. This task was recently undertaken in Ref. [3], under the assumption that there was a one-component uniform modified viscous fluid on the brane. The bulk viscosity ζ was taken to vary with the scalar expansion θ as $\zeta \propto \theta^{2\alpha-1}$, with α is a constant present in the 4D action

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} (f_0 R^\alpha + L_m). \quad (1)$$

Here $\kappa_4^2 = 8\pi G_4$, f_0 is a constant, and L_m is the Lagrangian of the matter. As shown in Ref. [4], when ζ varies in this way the formalism leads naturally to a Big Rip singularity in a finite future time. (This kind of theory is a natural generalization of the theory for a viscous Einstein fluid, $\alpha = 1$, as delineated in Ref. [5].) The main result of the investigation in Ref. [3] was that the 4D singularity on the brane becomes carried over to the 5D bulk, $|y| > 0$. The scale factors on the brane and in the bulk are actually proportional to each other.

The new element in the present investigation is that we consider a *two-fluid* model, with action

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} (R + f_0 R^\alpha + L_m). \quad (2)$$

It means that we include an Einstein fluid in addition to the modified fluid. There are physical motivations for this generalization. As emphasized by Vikman, for instance, [6], it is necessary to introduce a two-component model within the framework of the scalar field picture. There are moreover several recent works on cosmological models with two scalar fields; one may consult

Refs. [7, 8, 9, 10, 11, 12, 13], for instance. Equation (2) is better motivated physically than Eq. (1).

One may ask: is there the same close relationship in the two-fluid case between the singularities on the brane and in the bulk as in the single-fluid case? The answer turns out to be affirmative, as we shall show below.

Finally, we mention that there exist more general $f(R)$ theories. A general review of modified gravity can be found in Ref. [14], and recent reviews of $f(R)$ gravity theories unifying dark energy, inflation, and dark matter, can be found in Refs. [15, 16].)

2 A brief resumé of the 4D theory

We begin by reproducing some salient features of the 4D theory for a two-fluid system, following the treatment in Ref. [17]. The spatially flat FRW metric is

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad (3)$$

and the energy-momentum tensor of the viscous fluid is

$$T_{\mu\nu} = \rho U_\mu U_\nu + \tilde{p} h_{\mu\nu}, \quad (4)$$

where $h_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$ is the projection tensor and $\tilde{p} = p - \zeta\theta$ the effective pressure. In comoving coordinates, $U^0 = 1$, $U^i = 0$. From variation of the two-fluid action (2) we can derive the equations of motion (we put $\Lambda_4 = 0$). Of main interest is the (00)-component of the equations. We combine that particular component with the energy conservation law

$$\dot{\rho} + (\rho + p)3H = 9\zeta H^2, \quad (5)$$

which in turn is a consequence of the covariant conservation equation $\nabla^\nu T_{\mu\nu} = 0$. Here $H \equiv \dot{a}/a = \theta/3$ is the Hubble parameter. Some calculation leads to the equation

$$\begin{aligned} & 6\dot{H} + 9\gamma H^2 + \frac{3}{2}\gamma f_0 R^\alpha - 3\alpha f_0 [(3\gamma - 2)\dot{H} + 3\gamma H^2] R^{\alpha-1} \\ & + 3\alpha(\alpha - 1)f_0 [(3\gamma - 1)H\dot{R} + \ddot{R}] R^{\alpha-2} + 3\alpha(\alpha - 1)(\alpha - 2)f_0 \dot{R}^2 R^{\alpha-3} = 9\kappa_4^2 \zeta H, \end{aligned} \quad (6)$$

which is a nonlinear differential equation for $H(t)$ in view of the relationship $R = 6(\dot{H} + 2H^2)$.

Equation (6) is complicated. We will look for solutions of the form

$$H = \frac{H_*}{X}, \quad \text{where} \quad X \equiv 1 - BH_* t. \quad (7)$$

Here B is a nondimensional constant, whose value has to be calculated on the basis of initial assumptions for w, α , and ζ for the two fluid components. The star subscript refers to the initial (present) time $t_* = 0$. If $B > 0$, H will diverge in a finite singularity time $t = t_s$, and Big Rip will occur.

Assume now that the total bulk viscosity ζ is a sum of two parts, one part ζ_E referring to the Einstein fluid and a second part ζ_α referring to the modified fluid. As mentioned above, they will be taken to be proportional to θ and $\theta^{2\alpha-1}$, respectively. The corresponding proportionality factors are denoted τ_E and τ_α . Thus

$$\zeta_E = 3\tau_E H, \quad \zeta_\alpha = \tau_\alpha (3H)^{2\alpha-1}. \quad (8)$$

A consequence of these assumptions is that Eq. (6) becomes satisfied for the Einstein component and the modified component separately; the time-dependent terms automatically drop out. It turns out that there is a relationship between the factors τ_E and τ_α , the form of $\tau_\alpha = \tau_\alpha(\tau_E)$ being determined from α and f_0 as well as from w . The energy conservation law (5) holds for each component separately. From these equations we get two different expressions for the constant B :

$$B = -\frac{3\gamma}{2} + \frac{27\tau_E}{2} \frac{H_*^2}{\rho_{*E}}, \quad (9)$$

$$B = -\frac{3\gamma}{2\alpha} + \frac{3\tau_\alpha}{2\alpha} \frac{(3H_*)^{2\alpha}}{\rho_{*\alpha}}. \quad (10)$$

We have here for $t = 0$ the relations $\rho_* = \rho_{*E} + \rho_{*\alpha}$; moreover

$$\zeta_\alpha = \tau_\alpha \left(\frac{3H_*}{X} \right)^{2\alpha-1}, \quad \rho_E = \frac{\rho_{*E}}{X^2}, \quad \rho_\alpha = \frac{\rho_{*\alpha}}{X^{2\alpha}}. \quad (11)$$

3 Implications for the 5D theory

Now move on to consider the RS flat brane situated at $y = 0$, surrounded by an AdS space with metric

$$ds^2 = -n^2(t, y)dt^2 + a^2(t, y)\delta_{ij}dx^i dx^j + dy^2. \quad (12)$$

Here $n(t, y)$ and $a(t, y)$ are determined from Einstein's equations

$$R_{AB} - \frac{1}{2}g_{AB}R + g_{AB}\Lambda = \kappa^2 T_{AB}, \quad (13)$$

where $\kappa^2 = 8\pi G_5$. Note that whereas we put the 4D cosmological constant equal to zero above, we keep the 5D cosmological constant $\Lambda (< 0)$ different from zero in order to comply with the main idea of the RS model. The coordinates are $x^A = (t, x^1, x^2, x^3, y)$. From the junction conditions across the brane we get for arbitrary y , after integration with respect to y [3],

$$\left(\frac{\dot{a}}{na}\right)^2 = \frac{1}{6}\Lambda + \left(\frac{a'}{a}\right)^2 + \frac{C}{a^4}. \quad (14)$$

On the brane we may take $n_0(t) = 1$. Omitting the unimportant C term (the "radiation term"), we obtain then on the brane

$$H_0^2 = \frac{1}{6}\Lambda + \frac{\kappa^4}{36}(\sigma + \rho)^2, \quad (15)$$

where σ is the brane tension, assumed constant. We let generally the subscript zero refer to the brane.

The 5D equation (15) is important in our context, as it may be regarded to represent a bridge between the 4D and 5D cosmologies. Namely, by inserting for $\rho = \rho_E + \rho_\alpha$ from Eq. (11), we get

$$H_0^2 = \frac{1}{6}\Lambda + \frac{\kappa^4}{36} \left[\sigma + \frac{\rho_{*E}}{(1 - BH_* t)^2} + \frac{\rho_{*\alpha}}{(1 - BH_* t)^{2\alpha}} \right]^2. \quad (16)$$

We shall investigate the behavior of the scale factor near the Big Rip, occurring at $t_s = 1/(BH_*)$. We then assume that B is a positive quantity. We get approximatively

$$\frac{\dot{a}_0}{a_0} = \frac{\kappa^2}{6} \left[\frac{\rho_{*E}}{(1 - BH_* t)^2} + \frac{\rho_{*\alpha}}{(1 - BH_* t)^{2\alpha}} \right], \quad (17)$$

showing that the constants Λ and σ are no longer important in this limit. The solution is of the form

$$a_0(t) \sim \exp \left[\frac{(\kappa^2/6)\rho_{*E}}{(BH_*)^2(t_s - t)} + \frac{(\kappa^2/6)\rho_{*\alpha}}{(2\alpha - 1)(BH_*)^{2\alpha}(t_s - t)^{2\alpha-1}} \right]. \quad (18)$$

Two different sub-classes need here to be distinguished:

(i) *The case $\alpha < 1$.* The influence from the modified fluid component then becomes subdominant near the Big Rip. The behavior of $a(t)$ is governed by the Einstein fluid component. It might seem natural to suggest that this is after all the most likely scenario in the late universe.

(ii) *The case $\alpha > 1$.* Then, the modified fluid component will dominate at the end, regardless of what was the initial ratio between ρ_{*E} and $\rho_{*\alpha}$ at the initial instant $t = 0$. Moreover, the strength of the singularity is seen to be larger than in the Einstein case, the strength increasing for larger input values of α .

Let us now go back to the 5D equation (14) in the bulk. Taking into account the relation

$$\dot{n}(t, y) = \frac{\dot{a}(t, y)}{\dot{a}_0(t)}, \quad (19)$$

which in turn is a consequence of there being no energy flux in the y direction from the brane $T_{ty} = 0$ (cf., for instance, Ref. [18]), we obtain the solution

$$a^2(t, y) = \frac{1}{2}a_0^2(t) \left[\left(1 + \frac{\kappa^4 \sigma^2}{6\Lambda} \right) + \left(1 - \frac{\kappa^4 \sigma^2}{6\Lambda} \right) \cosh(2\mu y) - \frac{\kappa^2 \sigma}{3\mu} \sinh(2\mu|y|) \right], \quad (20)$$

with $\mu = \sqrt{-\Lambda/6}$. (Recall that we have assumed $C = 0$, $k = 0$.) This is formally the same solution as in the single-fluid case [3]. The characteristic properties of the two-fluid system turns up in the prefactor $a_0(t)$, but they do not influence the variation of $a(t, y)$ upon y in the bulk.

We may thus summarize our work as follows:

- The Big Rip singularity on the brane as following from the 4D theory - regardless of whether it is the Einstein fluid or the modified fluid that is the dominant component - becomes immediately transferred to the bulk, $|y| > 0$. The brane tension σ and the 5D cosmological constant Λ play no role for the establishment of the brane singularity, but they turn up again in the dependence of $a(t, y)$ upon y in the bulk region; cf. Eq. (20). The reason for this immediate effect is not known, but may be related to quantum mechanics.

- Our basic action (2) with α constant means a simple example of modified gravity. We assumed here a straightforward superposition of an Einstein fluid and a modified fluid. (More complicated modified gravity theories can be found, for example, in Refs. [14, 15, 16].) An important point in our analysis was to assume both fluid components viscous, and to assume the specific forms given in Eq. (8) for the bulk viscosities. These specific forms admit both fluid components to pass through the $w = -1$ barrier into the phantom region, and thereafter into the Big Rip singularity.

- The nature of the singularity on the brane was shown to depend on the magnitude of α . If $\alpha < 1$, the singularity near Big Rip was determined by the Einstein component, whereas for $\alpha > 1$ it was determined by the modified component. This kind of behavior was independent of the relative magnitude of the energy densities ρ_{*E} and $\rho_{*\alpha}$ at the initial instant $t = 0$. When the modified component dominates, the singularity is always stronger than in the Einstein case, and the singularity becomes stronger the higher is the value of α . As simple examples one may note that the value $\alpha = 1/2$, often considered, corresponding to a \sqrt{R} term in the Lagrangian, belongs to the Einstein case whereas the value $\alpha = 2$ (a R^2 term in the Lagrangian) belongs to the modified case.

- It should be emphasized that we have assumed the *bulk* viscosity only, simply omitting the shear viscosity which is known to be the most important component in ordinary hydrodynamics. This seems to be a natural way to proceed, all the time that we are assuming spatial isotropy in the cosmic fluid.

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